



THE UNIVERSITY
of LIVERPOOL

JANUARY 2006 EXAMINATIONS

Degree of Bachelor of Science: Year 3
Degree of Master of Physics: Year 4

STATISTICAL AND LOW TEMPERATURE PHYSICS

TIME ALLOWED: THREE HOURS

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

Question 1 carries 50% of the total marks.

Question 2 and 3 each carry 25% of the total marks.

The marks allotted to each part of a question are indicated in square brackets.

In the event of a student answering both parts of an either/or question and not clearly crossing out one answer, only the answer to part (a) of the question will be marked.

1.

(a). A set of 4 distinguishable particles occupies energy states $0, \epsilon, 2\epsilon, 3\epsilon, 4\epsilon, \dots$

The total energy of the set is 5ϵ .

- (i) Write out the 6 possible distributions [2]
- (ii) Evaluate the number of microstates for each distribution [2]
- (iii) Evaluate the mean populations of the states. [2]

(b) N atoms bound into a solid system at temperature T can each exist in states of energy $0, \epsilon, 2\epsilon$.

- (i) Write an expression for the Partition function of the atoms. [2]
- (ii) Using the bridge relation

$$U = NkT^2 \frac{\partial (\ln Z)}{\partial T}$$

or otherwise, show that the internal energy U can be written as

$$U = \frac{N\epsilon \exp(-\epsilon/kT) [1 + 2\exp(-\epsilon/kT)]}{[1 + \exp(-\epsilon/kT) + \exp(-2\epsilon/kT)]} \quad [2]$$

- (iii) Derive the limiting values of U as $T \rightarrow 0$ and as $T \rightarrow \infty$. [2]
- (iv) Sketch a graph of U versus T. [2]
- (v) Without further differentiation sketch the graph of C_V versus T. [2]

(c) The Maxwell-Boltzmann distribution of speeds of molecules $n(v)$ in a box containing N molecules of mass m at temperature T is written

$$n(v) = 4\pi N \left(\frac{m}{2\pi kT} \right)^{3/2} v^2 \exp(-mv^2/2kT)$$

- (i) Draw the distribution $n(v)$ versus v. [2]
- (ii) Derive an expression for the most probable velocity v_p . [2]
- (iii) Derive an expression for the mean square speed v_m^2 . [2]
- (iv) Estimate the energy of a mole (6×10^{23} molecules) of a monatomic gas at room temperature (300K). [2]

$$\text{Integrals } I_n = \int_0^\infty x^n \exp(-bx^2)$$

Can be evaluated from

$$I_n = [(n-1)/2b] I_{n-2}, \quad I_0 = \frac{1}{2}(\pi/b)^{1/2}, \quad I_1 = 1/2b$$

(d) The molecule HD is composed of an atom of hydrogen (H, atomic mass = 1) bonded to an atom of deuterium (D, atomic mass = 2).

Vibrational energy levels of such molecules are given by

$$E(n) = (n + \frac{1}{2}) h (B/\mu)^{\frac{1}{2}}$$

where B is the inter-atomic force constant, μ is the reduced mass of the molecule and n is an integer.

Rotational energy levels are given by the relation

$$E(I) = \frac{h^2 I(I+1)}{2\mu r^2}$$

where the integer I is the quantised angular momentum and r is the separation of the atoms.

(i) Evaluate excitation energies for the first excited states of vibrational and rotational motion for the HD molecule in which $B = 6.91 \times 10^2 \text{ kg.s}^{-1}$ and $r = 1.05 \times 10^{-10} \text{ m}$. [2]

(ii) Estimate temperatures θ_V and θ_R at which vibrational and rotational motions become populated for the HD molecule. [2]

(iii) Give reasoned estimates for the heat capacity C_V of a mole of HD at temperatures $T = 20 \text{ K}$ and at $T = 300 \text{ K}$. [2],[2]

(e)

(i) Draw diagrams depicting the constituents of an atom of He^3 and an atom of He^4 . [2]

(ii) In each case specify the electronic, nuclear and total angular momentum values of the atom [2]

(iii) Explain why one type of atom is a fermion and the other is a boson. [2]

(iv) He^3 atoms in liquid form at $T = 2 \text{ K}$ in a container occupy a level scheme of quantised states. Sketch the occupation of these levels. [2]

(v) He^4 atoms in liquid form at $T = 2 \text{ K}$ in a container occupy a level scheme of quantised states. Sketch the occupation of these levels. [2]

(f) The critical field $B_C(T)$ in a superconductor at temperature T is related to that at $T = 0$, $B_C(0)$, by the relation

$$B_C(T) = B_C(0) \cdot [1 - (T/T_C)^2]$$

where T_C is the critical temperature at $B = 0$.

(i) Sketch the relation $B_C(T)$ versus T. [2]

(ii) Label the regions of superconducting and normal behaviour on the sketch [2]

(iii) The element lead has $T_C = 7.2 \text{ K}$ and $B_C(0) = 0.080 \text{ T}$. Calculate the critical temperature for lead in a field of $B = 0.05 \text{ T}$. [2]

(iv) Is lead at a temperature of $T = 5.6 \text{ K}$ in an applied field of $B = 0.035 \text{ T}$ in a superconducting or a normal state? [2]

2. Answer **either** 2(a) **or** 2(b)

2(a) A system is composed of N conduction electrons which move freely within a cube of metal of side L .

(i) Write quantised values for the wavevector components k_x, k_y, k_z corresponding to allowed motions of the electrons. [2]

(ii) Illustrate the allowed states in k_x, k_y, k_z space. [2]

(iii) Taking account of the spin degeneracy of electrons, show that the number of states $g(k)$ with values of k in the range k to $k + dk$ is

$$g(k)dk = \frac{2 \cdot V \cdot 4\pi k^2 \cdot dk}{(2\pi)^3}$$

where the volume $V = L^3$. [4]

(iv) Write the relation between k and energy ϵ . [1]

(v) Show that the number of states with energy in the range ϵ to $\epsilon + d\epsilon$ is

$$g(\epsilon)d\epsilon = \frac{2 \cdot V \cdot (2m/\hbar^2)^{3/2} \cdot \epsilon^{1/2} \cdot d\epsilon}{(2\pi)^2} \quad [3]$$

(vi) The probability that a state with energy ϵ will be occupied is $f(\epsilon)$.

Sketch graphs of $f(\epsilon)$ versus ϵ for electrons

For temperature $T = 0$

For temperature $0 < T < T_F$ where T_F is the Fermi temperature.

Indicate the Fermi energy $\mu(0)$ at $T = 0$. [3]

(vii) Show that $\mu(0)$ is given by

$$\mu(0) = (\hbar^2/2m) \cdot (3\pi^2 N/V)^{2/3} \quad [4]$$

At temperature T the internal energy U of the electrons can be written

$$U = \frac{2N\mu(0)}{3} + \frac{N\pi^2 \cdot (kT)^2}{4\mu(0)}$$

Metallic silver has a molar volume of $10.27 \times 10^{-6} \text{ m}^3$ and each atom contributes one electron to the conduction band.

(viii) Evaluate the Fermi energy $\mu(0)$ for silver. [3]

(ix) Evaluate the electron heat capacity C_V for a molar quantity of silver at a temperature of 5K. [3]

2(b)

(i) Describe what is meant by black body radiation.

[4]

The density of states $g(k)$ in terms of wave-vector k for quantised electromagnetic waves in a cavity of volume V is written as

$$g(k)dk = \frac{2 \cdot V \cdot 4\pi k^2 dk}{(2\pi)^3}$$

(ii) Show that this density of states can be written in terms of the frequency ν of the waves as

$$g(\nu)d\nu = \frac{8\pi V \nu^2 d\nu}{c^3}$$

where c is the velocity of light.

[3]

(iii) The energy contained in the frequency interval ν to $\nu + d\nu$ of the radiation is given by

$$\epsilon(\nu)d\nu = \frac{8\pi V \nu^2 d\nu}{c^3} \cdot h\nu \cdot \frac{1}{[\exp(h\nu/kT) - 1]}$$

Explain the meaning of the factors $h\nu$ and $\frac{1}{[\exp(h\nu/kT) - 1]}$ in this expression.

[2]

(iv) Deduce the limiting values of $\epsilon(\nu)$ as $\nu \rightarrow 0$ and as $\nu \rightarrow \infty$.

[2],[2]

(v) Sketch the distribution of $\epsilon(\nu)$ versus ν and relate the energy density (U/V) to the sketch.

[4]

(vi) Using the integral given below – show that

$$(U/V) = \frac{8\pi^5 \cdot (kT)^4}{15 \cdot (hc)^3}$$

[4]

(vii) Evaluate the energy density at $T = 1000K$

[2]

(viii) Calculate the temperature at which the energy density is 10 times greater than it is at $T = 1000K$.

[2]

$$\frac{y^3 dy}{[\exp(y) - 1]} = \frac{\pi^4}{15}$$

Answer **either** 3(a) **or** 3(b).

3(a)

(i) Describe the basic features of the microscopic theory of superconductivity. [4]

Use this theory to explain:

(ii) the nature of the current carriers in the normal and superconducting states [2]

(iii) the critical temperature T_C and the critical field H_C of a superconductor [3]

(iv) the isotope effect [3]

(v) the comparison between the resistivity of a non-superconducting metal (Cu) and the normal phase of a superconducting metal (Pb). [3]

(vi) the nature of high T_C superconductors [3]

(vi) Describe the Meissner effect. [3]

(viii) Draw graphs to illustrate the behaviour of Type I and Type II superconductors [2]

(ix) Give an example of a Type I and a Type II superconductor. [2]

3(b)

Draw the pressure versus temperature phase diagram for He^4 and label the different phases. [4]

Describe a model for liquid He^4 and show how this model explains the measurements viscosity for this liquid phase. [4]

Draw the pressure versus temperature phase diagram for He^3 and label the different phases. [4]

Does liquid He^3 show superfluid properties? If so give a brief description of how these may be explained. [2]

Why may parts of the phase diagram be sensitive to the application of a magnetic field? [2]

Describe **one** method of attaining temperatures of about 1mK using liquid helium.

The description should include the type of helium used, the basic theory of the method, a schematic sketch of the apparatus and the starting temperature of the process.

[9]